# RESONANT CHARACTERISTICS OF DIELECTRIC RESONATORS FOR MILLIMETER-WAVE INTEGRATED CIRCUITS*

Tatsuo Itoh and Chen Chang
Department of Electrical Engineering
University of Kentucky
Lexington, KY 40506

ABSTRACT

An approximate but accurate method is presented for computing the characteristics of dielectric resonators used in millimeter-wave integrated circuits. Physical interpretation of the modal fields, numerical results and experimental verification are included.

## Introduction

This paper presents an approximate theory of predicting accurately the resonant frequencies of a resonator made of a section of rectangular dielectric waveguide shown in Fig. 1. This type of resonator has been used in millimeter-wave integrated circuits as a cavity of IMPATT or Gunn oscillators.[1,2] In a typical design method for such a resonator, one first computes the guide wavelength $\lambda_g$ of an infinitely long dielectric waveguide by using an approximate analysis such as the one developed by Marcatili,[3] and then the resonant frequency is computed by assuming the end surfaces of the resonator are magnetic walls, that is, open-circuited.[1]

The method developed in this paper is based on Marcatili's approximation that is extended here to apply for three dimensional resonator structures. A similar approach has been used for computing the resonant frequencies of the dominant mode in a pill-box resonator and the results so obtained agreed very well with data obtained from a more rigorous theory as well as experimental results.[4,5]

In a manner similar to that in Ref. 4, the present approach assumes a more realistic field distribution in resonator structures than the conventional magnetic wall method, and replaces the open-circuit condition with an impedance termination at the ends of the resonator. The limitation of the present method is identified and some numerical and experimental results presented.

## Method of Analysis

Fig. 2 shows the side and top views of a rectangular dielectric resonator. In an infinitely long dielectric waveguide, a finite number of guided modes exist which are classified into $E^y$ and $E^x$ modes.[3] Here we restrict ourselves to the $E^y$ modes only. According to Marcatili, the propagation constant of the $E^y_{pq}$ mode can be derived from

$$k_z = \sqrt{\varepsilon_r k_o^2 - k_x^2 - k_y^2} \qquad (1)$$

where $k_x$ and $k_y$ are the solutions of

$$k_x a = p\pi - 2\tan^{-1}\left(\frac{k_x}{\xi}\right) \qquad p = 1, 2, \cdots \qquad (2)$$

$$k_y b = q\pi - 2\tan^{-1}\left(\varepsilon_r \frac{k_y}{\eta}\right) \qquad q = 1, 2, \cdots \qquad (3)$$

$$\xi^2 = (\varepsilon_r - 1)k_o^2 - k_x^2$$

$$\eta^2 = (\varepsilon_r - 1)k_o^2 - k_y^2$$

The dominant field components are $E_y$ and $H_x$. Hence, in a finitely long dielectric waveguide to be used as a resonator, the field distributions are

$$E_y = E(x, y)e^{-jk_z z} + E'(x, y)e^{+jk_z z} \qquad (4)$$

$$H_x = -\frac{1}{Z_o}\left\{E(x, y)e^{-jk_z z} - E'(x, y)e^{+jk_z z}\right\} \qquad (5)$$

where $Z_o = \frac{\omega\mu}{k_z}$. Under the magnetic wall model, $H_x = 0$ at $z = \pm\ell$ which gives rise to the resonant frequencies of the $E^y_{pqn}$ mode

$$k_z \ell = \frac{n}{2}\pi \qquad n = 0, 1, 2, \cdots \qquad (6)$$

In this paper, we assume that the waveguide is terminated with impedance walls rather than magnetic walls. We will now derive such impedances. To this end, extending the Marcatili's theory, we assume that for a high Q resonator the fields in the shaded regions in Fig. 2 are negligible and hence we only need to match the field at the dielectric-air interfaces at $z = \pm\ell$. For $z > 0$, $|x| < a/2$ and $|y| < b/2$, the field may be written as

$$E_y = CE(x, y)e^{-\gamma(z-\ell)} \qquad (7)$$

$$H_x = -\frac{1}{Z_L} CE(x, y)e^{-\gamma(z-\ell)} \qquad (8)$$

$$Z_L = \frac{j\omega\mu}{\gamma}, \quad \gamma^2 = k_x^2 + k_y^2 - k_o^2 \qquad (9)$$

The field distributions in the x and y directions are assumed to be identical to those inside the waveguide. Matching $E_y$ and $H_x$ fields at $z = \pm\ell$ and taking into

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account the symmetry, we obtain eigenvalue equations for even and odd modes as follows:

$$k_z \tan k_z \ell = \gamma \qquad (10)$$

$$k_z \cot k_z \ell = -\gamma \qquad (11)$$

The equivalent circuit of the resonator may be given by Fig. 2(c). In the magnetic wall model, $\gamma = 0$ and hence $Z_L = \infty$ in (9).

Before attempting numerical solutions, we study the nature of (10) and (11). When

$$k_o^2 < k_x^2 + k_y^2 < \varepsilon_r k_o^2 \text{ or } (\varepsilon_r - 1)k_o^2 > k_z^2 \qquad (12)$$

$\gamma$ in (9) is real and, hence, within our approximation, the radiation from the structure is negligible and a high-Q resonator is obtained. When $(\varepsilon_r - 1)k_o^2 < k_z^2 < \varepsilon_r k_o^2$, radiation occurs from two end surfaces. This phenomenon may be qualitatively explained as follows: The modal field in the waveguide can be decomposed into a set of plane waves (rays) which are subject to total internal reflection at the side walls of the guide. At the end surfaces of the resonator, the incident angles of these rays are still large enough to cause total internal reflection if (12) is satisfied. Hence, the field outside of the resonator decays exponentially at the rate of $\gamma$. When $k_z$ is larger than $(\varepsilon_r - 1)k_o^2$, the direction of rays makes smaller angle with the z-axis, and the incident angle at the end surface becomes less than the critical angle of total internal reflection, and the radiation occurs as $\gamma$ becomes imaginary. Therefore, even though the waveguide mode is better guided, the situation is not good for the resonator application.

## Results

Fig. 3 shows computed results of resonant characteristics for the dominant even ($E^y_{110}$) and odd ($E^y_{111}$) modes. The results obtained from the magnetic wall model are also plotted for $E^y_{111}$. It is seen that agreement between the two results for $E^y_{111}$ is not necessarily good unless the frequency is high enough so that $\gamma \simeq 0$. Note also that the magnetic wall model for the even dominant mode becomes trivial.

In Fig. 4, the approximate $E_y$ field distribution is plotted for dominant even and odd modes. In most millimeter-wave IC applications such as the oscillators, it is more practical to use $E^y_{111}$ and higher order modes because $E^y_{110}$ mode requires the length $2\ell$ to be extremely small. This situation is in contrast to MIC application where a pill-box resonator in the dominant even mode is used.

Fig. 5 presents comparison of computed and measured results of resonant frequencies. The measured results are closer to the data computed in this paper by using the impedance wall model than those from the magnetic wall model.

## Conclusions

We presented a more rigorous analysis than conventional magnetic wall methods for analyzing the resonant properties of dielectric resonators to be used in millimeter-wave integrated circuits. Some qualitative explanation was given for the resonant phenomena and numerical results based on the new method have been presented and compared with experimental data. The results are believed to be useful for a number of applications in millimeter-wave integrated circuits.

## References


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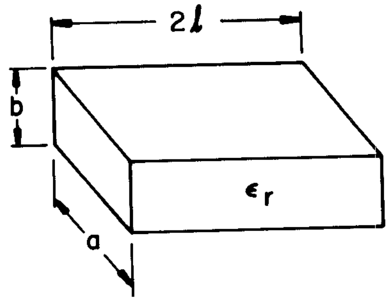


Fig. 1 Dielectric resonator

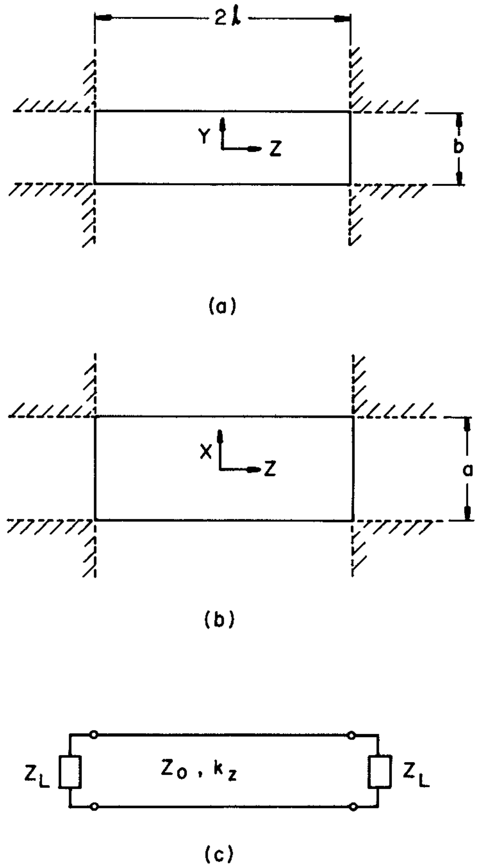


Fig. 2. Side and top views and equivalent circuit of the resonator

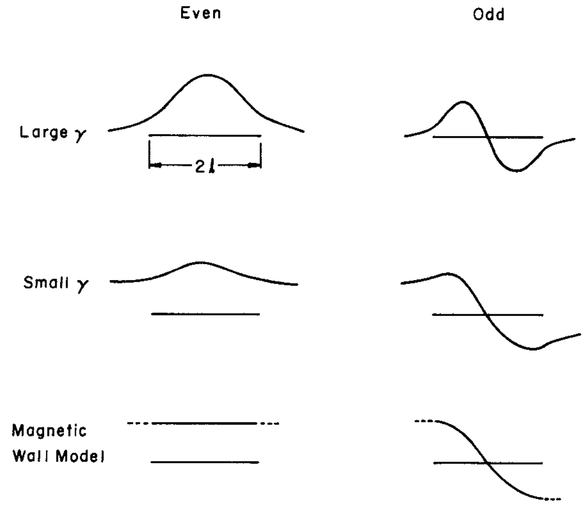


Fig. 4. $E_y$ field distributions in the resonator

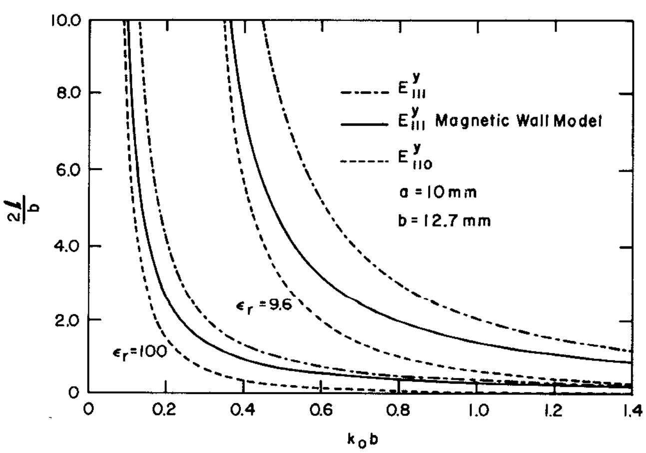


Fig. 3. Resonant characteristics

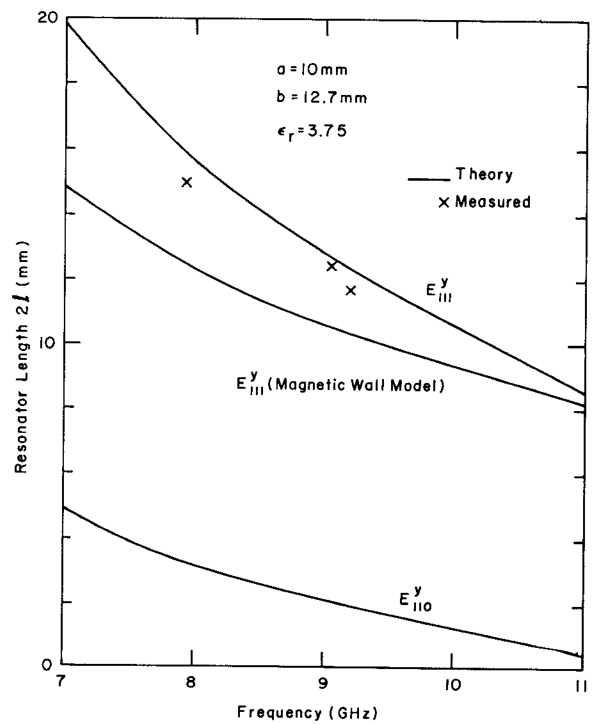


Fig. 5. Resonant frequency versus the resonator length